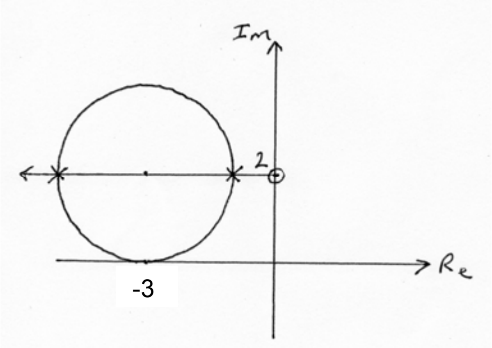
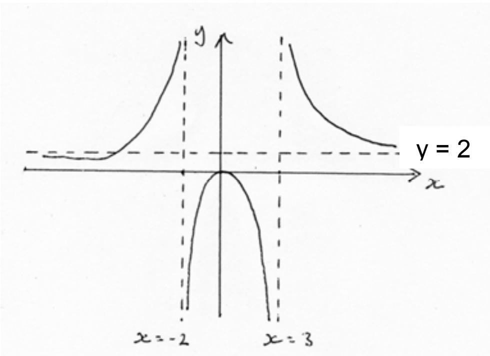
4755 (FP1) Further Concepts for Advanced Mathematics

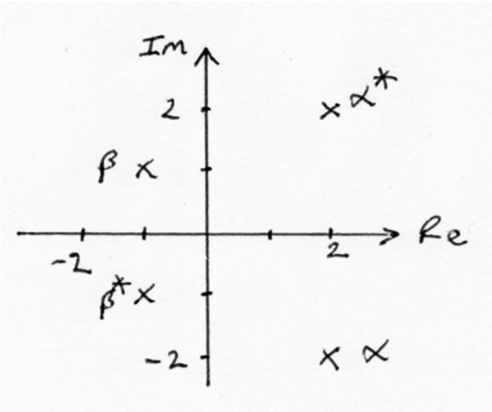
Qu	Answer	Mark	Comment
Section A			
1(i)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	B1	
1(ii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1	
1(iii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$	M1 A1 **[4]**	Multiplication, or other valid method (may be implied) c.a.o.
2	 	B3 B3 B1 **[7]**	Circle, B1; centre $-3+2\text{j}$, B1; radius = 2, B1 Line parallel to real axis, B1; through (0, 2), B1; correct half line, B1 Points $-1+2\text{j}$ and $-5+2\text{j}$ indicated c.a.o.
3	$\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow -x-y=x,\ 2x+2y=y$ $\Rightarrow y=-2x$	M1 M1 B1 **[3]**	For $\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$
4	$3x^3 - x^2 + 2 \equiv A(x-1)^3 + (x^3 + Bx^2 + Cx + D)$ $\equiv Ax^3 - 3Ax^2 + 3Ax - A + x^3 + Bx^2 + Cx + D$ $\equiv (A+1)x^3 + (B-3A)x^2 + (3A+C)x + (D-A)$ $\Rightarrow A=2,\ B=5,\ C=-6,\ D=4$	M1 B4 **[5]**	Attempt to compare coefficients One for each correct value

5(i)	$\mathbf{AB}=\begin{pmatrix}7&0&0\\0&7&0\\0&0&7\end{pmatrix}$	B3 **[3]**	Minus 1 each error to minimum of 0
5(ii)	$\mathbf{A}^{-1}=\frac{1}{7}\begin{pmatrix}-1&0&2\\14&-14&7\\-5&7&-4\end{pmatrix}$	M1 A1 **[2]**	Use of B c.a.o.
6	$w=2x\Rightarrow x=\frac{w}{2}$ $\Rightarrow 2\left(\frac{w}{2}\right)^3+\left(\frac{w}{2}\right)^2-3\left(\frac{w}{2}\right)+1=0$ $\Rightarrow w^3+w^2-6w+4=0$	B1 M1 A1 A2 **[5]**	Substitution. For substitution $x=2w$ give B0 but then follow through for a maximum of 3 marks Substitute into cubic Correct substitution Minus 1 for each error (including '= 0' missing), to a minimum of 0 Give full credit for integer multiple of equation
6	**OR** $\alpha+\beta+\gamma=-\frac{1}{2}$ $\alpha\beta+\alpha\gamma+\beta\gamma=-\frac{3}{2}$ $\alpha\beta\gamma=-\frac{1}{2}$ Let new roots be *k*, *l*, *m* then $k+l+m=2(\alpha+\beta+\gamma)=-1=\frac{-B}{A}$ $kl+km+lm=4(\alpha\beta+\alpha\gamma+\beta\gamma)=-6=\frac{C}{A}$ $klm=8\alpha\beta\gamma=-4=\frac{-D}{A}$ $\Rightarrow \omega^3+\omega^2-6\omega+4=0$	B1 M1 A1 A2 **[5]**	All three Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation Sums and products all correct ft their coefficients; minus one for each error (including '= 0' missing), to minimum of 0 Give full credit for integer multiple of equation

7(i)	$\frac{1}{3r-1}-\frac{1}{3r+2}\equiv\frac{3r+2-(3r-1)}{(3r-1)(3r+2)}$	M1	Attempt at correct method
	$\equiv\frac{3}{(3r-1)(3r+2)}$	A1	Correct, without fudging
		[2]	
7(ii)	$\sum_{r=1}^{n}\frac{1}{(3r-1)(3r+2)}=\frac{1}{3}\sum_{r=1}^{n}\left[\frac{1}{3r-1}-\frac{1}{3r+2}\right]$	M1	Attempt to use identity
	$=\frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+.......+\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)\right]$	A1 M1	Terms in full (at least two) Attempt at cancelling
	$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{3n+2}\right]$	A2	A1 if factor of $\frac{1}{3}$ missing,
		[5]	A1 max if answer not in terms of n
			Section A Total: 36

Section B

8(i)	$x = 3$, $x = -2$, $y = 2$	B1 B1 B1 **[3]**	
8(ii)	Large positive *x*, $y \to 2^+$ (e.g. consider $x = 100$) Large negative *x*, $y \to 2^-$ (e.g. consider $x = -100$)	M1 B1 B1 **[3]**	Evidence of method required
8(iii)	Curve Central and RH branches correct Asymptotes correct and labelled LH branch correct, with clear minimum  	B1 B1 B1 **[3]**	
8(iv)	$-2 < x < 3$ $x \neq 0$	B2 B1 **[3]**	B2 max if any inclusive inequalities appear B3 for $-2 < x < 0$ and $0 < x < 3$,

9(i)	$2+2\mathrm{j}$ and $-1-\mathrm{j}$	B2 **[2]**	1 mark for each
9(ii)		B2 **[2]**	1 mark for each correct pair
9(iii)	$(x-2-2\mathrm{j})(x-2+2\mathrm{j})(x+1+\mathrm{j})(x+1-\mathrm{j})$ $=(x^2-4x+8)(x^2+2x+2)$ $=x^4+2x^3+2x^2-4x^3-8x^2-8x+8x^2+16x+16$ $=x^4-2x^3+2x^2+8x+16$ $\Rightarrow A=-2,\ B=2,\ C=8,\ D=16$	M1 B2 A1 M1 A2 **[7]**	Attempt to use factor theorem Correct factors, minus 1 each error B1 if only errors are sign errors One correct quadratic with real coefficients (may be implied) Expanding Minus 1 each error, A1 if only errors are sign errors
	OR $\sum\alpha=2$ $\alpha\beta\gamma\delta=16$ $\sum\alpha\beta=\alpha\alpha^*+\alpha\beta+\alpha\beta^*+\beta\beta^*+\beta\alpha^*+\beta^*\alpha^*$ $\sum\alpha\beta\gamma=\alpha\alpha^*\beta+\alpha\alpha^*\beta^*+\alpha\beta\beta^*+\alpha^*\beta\beta^*$ $\sum\alpha\beta=2$, $\sum\alpha\beta\gamma=-8$ $A=-2,\ B=2,\ C=8,\ D=16$	B1 B1 M1 M1 A1 A2 **[7]**	 Both correct Minus 1 each error, A1 if only errors are sign errors
	OR Attempt to substitute in one root Attempt to substitute in a second root Equating real and imaginary parts to 0 Attempt to solve simultaneous equations $A=-2,\ B=2,\ C=8,\ D=16$	M1 M1 A1 M1 M1 A2 **[7]**	 Both correct Minus 1 each error, A1 if only errors are sign errors

Qu	Answer	Mark	Comment
Section B (continued)			
10(i)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	M1	Separation of sums (may be implied)
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	B1	One mark for both parts
	$= \frac{1}{12}n(n+1)\left[3n(n+1)+2(2n+1)\right]$	M1	Attempt to factorise (at least two linear algebraic factors)
	$= \frac{1}{12}n(n+1)(3n^2+7n+2)$	A1	Correct
	$= \frac{1}{12}n(n+1)(n+2)(3n+1)$	E1	Complete, convincing argument
		[5]	
10(ii)	$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(3n+1)$		
	n = 1, LHS = RHS = 2	B1	2 must be seen
	Assume true for $n = k$	E1	Assuming true for k
	$\sum_{r=1}^{k} r^2(r+1) = \frac{1}{12}k(k+1)(k+2)(3k+1)$		
	$\sum_{r=1}^{k+1} r^2(r+1)$		
	$= \frac{1}{12}k(k+1)(k+2)(3k+1)+(k+1)^2(k+2)$	B1	(k + 1)th term
	$= \frac{1}{12}(k+1)(k+2)\left[k(3k+1)+12(k+1)\right]$	M1	Attempt to factorise
	$= \frac{1}{12}(k+1)(k+2)(3k^2+13k+12)$	A1	Correct
	$= \frac{1}{12}(k+1)(k+2)(k+3)(3k+4)$	A1	Complete convincing argument
	$= \frac{1}{12}(k+1)((k+1)+1)((k+1)+2)(3(k+1)+1)$	E1	Dependent on previous A1 and previous E1
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for k = 1, 2, 3 and so true for all positive integers.	E1	Dependent on first B1 and previous E1
		[8]	
			Section B Total: 36
			Total: 72